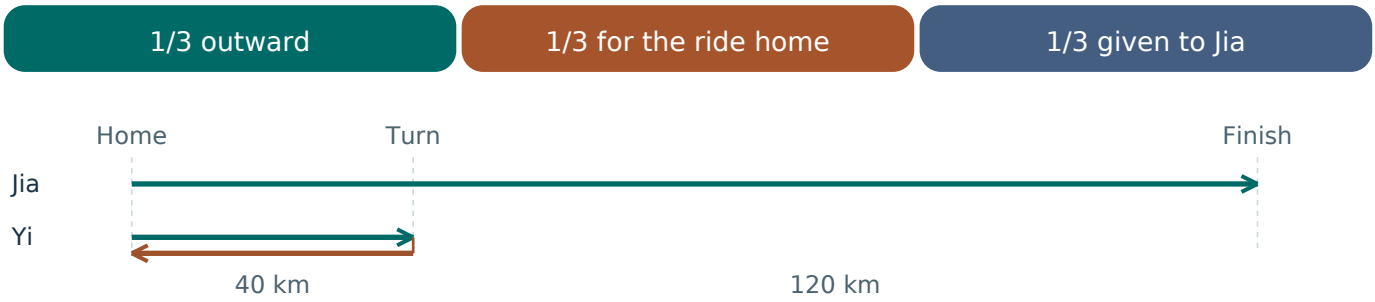
How far can the delivery bike go?

One full tank carries any bike 120 km. The same fraction of a tank takes each bike the same distance.

Jia (甲) carries the parcel. Yi (乙) and Bing (丙) are the helpers.

01 Two bikes: split Yi’s tank into thirds



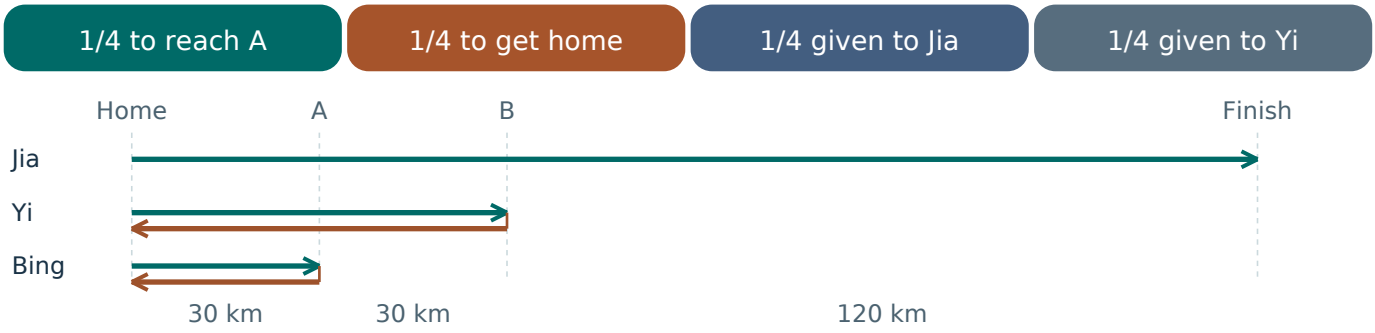
1 Yi keeps 1/3 tank for the ride home. The outward route has the same length, so each bike has already used 1/3 tank to reach the turn.

2 The turn is 120 × 1/3 = 40 km from home. Yi gives Jia the remaining 1/3 tank, which exactly fills the space in Jia’s tank.

3 Jia now rides another 120 km on a full tank. Count both legs of Jia’s ride.

Q1 / Q2 **1/3 + 1 = 4/3 tanks · 40 + 120 = 160 km**

02 Three bikes: keep fuel for the whole return



1 To A, each bike uses 1/4 tank and rides 30 km. Bing fills Jia and Yi, then returns.

2 Yi sets aside 1/4 tank for A to home. Divide the remaining 3/4 equally between A to B, B back to A, and topping up Jia. Each share is (3/4) ÷ 3 = 1/4.

3 B is 60 km from home. Yi keeps 1/2 tank to return. Jia leaves B with a full tank.

Q3 **1/4 + 1/4 + 1 = 3/2 tanks**

Eleven bikes, ten turnarounds

Choose one fuel unit small enough to refill everyone at the first stop.

03 Make the first stop 10 km from home

1 The first helper supplies ten other bikes. Its tank must cover its own outward ride, its own return, and ten top-ups. Each top-up replaces exactly what one bike burned on the outward ride.

2 These are 12 equal shares. Each share is 1/12 tank and takes a bike 120 ÷ 12 = 10 km. Call this one share a block.

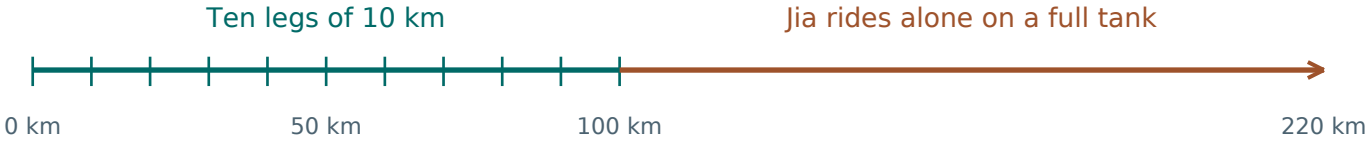


One helper’s full tank: two blocks for its own ride and ten for the continuing bikes.

04 Use the same 10 km spacing again

1 At 20 km, the second helper has used one block since its last refill. It needs two blocks to reach home and can give one block to each of the nine other bikes. The total is 1 + 2 + 9 = 12.

2 At stop k, the helper uses one block on the latest leg, keeps k blocks for home, and gives one block to each of the 11 − k remaining bikes. The sum is always 1 + k + (11 − k) = 12. Every continuing bike leaves full.



Q4 **10 × 10 + 120 = 220 km**

With n bikes, use n + 1 equal blocks per tank. There are n − 1 stops spaced 120/(n + 1) km apart. The last 120 km are Jia’s alone.

Can enough bikes reach 260 km?

A bound on the last helper’s return gives the answer without counting the bikes.

05 Look at the final transfer

1. The last helper must ride directly home with the fuel it keeps. Even a full tank would take it only 120 km, so the last transfer is at most 120 km from home.

2. After that transfer Jia can travel at most 120 km. Thus the destination is at most 120 + 120 = 240 km from home.

Q5 **No. 260 km is farther than 240 km.**

06 Why is the equal-spacing plan optimal?

Optional proof. Use 120 km as one distance unit, so a bike burns one tank per unit. Number the return points from nearest to farthest. Write x_j for the distance from home to stop j and S_j for the sum of the first j return distances.

1. At stop j, the first j helpers have used or reserved $2S_j$ tanks for their round trips. Each of the other $n - j$ bikes has travelled x_j and can hold at most one tank. Accounting for all n starting tanks gives

$$n \le 2S_j + (n - j)x_j + (n - j).$$

2. Put $x_j = S_j - S_{j-1}$ into the inequality and collect the terms:

$$(n - j + 2)S_j \ge j + (n - j)S_{j-1}.$$

Start at $S_0 = 0$. A larger previous sum can only raise the lower bound on the next sum. Equal spacing gives equality at each stop because the continuing bikes leave full. Step by step, this plan attains the smallest possible sum.

3. All bikes finish empty, so Jia’s distance in these units is $n - 2S_{n-1}$. The smallest total return distance therefore gives the longest delivery ride.

RANGE **$120 + (n - 1) \times 120/(n + 1) = 240n/(n + 1)$ km**